CHAPTER 2

Applications of Integrals

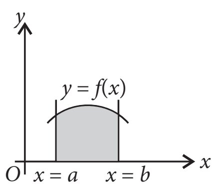
Recap Notes

INTRODUCTION

In geometry, we have learnt formulae to calculate areas of various geometrical figures. Such formulae of elementary geometry allow us to calculate areas of many simple figures. However, they are inadequate for calculating the areas enclosed by curves. For that we shall need some concepts of integral calculus.

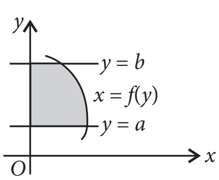
Area Under Simple Curves

- Area of shaded portion, as shown in figure, is given by



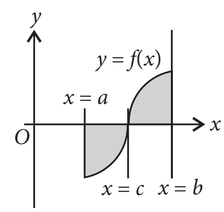
$$A = \int_a^b f(x)dx$$

- Area of shaded portion, as shown in figure, is given by



$$A = \int_a^b f(y)dy$$

- Area of shaded portion, as shown in figure, is given by



$$A = \left|\int_a^c f(x)\,dx\right| + \int_c^b f(x)\,dx$$

- The area of a region bounded by $y^2 = 4ax$ and $x^2 = 4by$ is $\frac{16ab}{3}$ sq. units.
- The area of a region bounded by $y^2 = 4ax$ and $y = mx$ is $\frac{8a^2}{3m^3}$ sq. units.
- The area of a region bounded by $y^2 = 4ax$ and its latus rectum is $\frac{8a^2}{3}$ sq. units.
- The area of a region bounded by one arc of $\sin ax$ or $\cos ax$ and x-axis is $\frac{2}{a}$ sq. units.
- Area of region bounded by an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab sq. units.
- The area of a region bounded by $y = ax^2 + bx + c$ and x-axis is $\frac{(b^2 - 4ac)^{\frac{3}{2}}}{6a^2}$ sq. units.

Practice Time

OBJECTIVE TYPE QUESTIONS

Multiple Choice Questions (MCQs)

1. The area bounded by the curve $y = x^2 + 4x + 5$, the axes of coordinates and minimum ordinate is

(a) $3\frac{2}{3}$ sq. units (b) $4\frac{2}{3}$ sq. units

(c) $5\frac{2}{3}$ sq. units (d) none of these

2. Area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

(a) $4\pi ab$ sq. units (b) $2\pi ab$ sq. units

(c) πab sq. units (d) $\frac{\pi ab}{2}$ sq. units

3. The area bounded by the curve $2x^2 + y^2 = 2$ is

(a) π sq. units (b) $\sqrt{2}\pi$ sq. units

(c) $\frac{\pi}{2}$ sq. units (d) 2π sq. units

4. Area enclosed by the circle $x^2 + y^2 = a^2$ is equal to

(a) $2\pi a^2$ sq. units (b) πa^2 sq. units

(c) $2\pi a$ sq. units (d) πa sq. units

5. Area bounded by the ellipse $\frac{x^2}{4}+\frac{y^2}{9} = 1$ is

(a) 6π sq. units (b) 3π sq. units

(c) 12π sq. units (d) none of these

6. The area enclosed between the curve $x^2 + y^2 = 16$ and the coordinate axes in the first quadrant is

(a) 4π sq. units (b) 3π sq. units

(c) 2π sq. units (d) π sq. units

7. The area enclosed by the curve $\frac{x^2}{25}+\frac{y^2}{9}=1$ is

(a) 10π sq. units (b) 15π sq. units

(c) 5π sq. units (d) 4π sq. units

8. The area bounded by the curve $y = f(x)$, the x-axis and $x = 1$ and $x = b$ is $(b - 1) \sin (3b + 4)$. Then, $f(x)$ is

(a) $(x - 1) \cos (3x + 4)$

(b) $\sin (3x + 4)$

(c) $\sin (3x + 4) + 3(x - 1)\cdot\cos (3x + 4)$

(d) none of these

9. The area of the region bounded by the parabola $y = x^2 + 1$ and the straight line $x + y = 3$ is given by

(a) $\frac{45}{7}$ sq. units (b) $\frac{25}{4}$ sq. units

(c) $\frac{5}{18}$ sq. units (d) $\frac{9}{2}$ sq. units

10. The area enclosed between the curve $y^2 = 4x$ and the line $y = x$ is

(a) $\frac{8}{3}$ sq. units (b) $\frac{4}{3}$ sq. units

(c) $\frac{2}{3}$ sq. units (d) $\frac{1}{2}$ sq. units

11. The area bounded by the lines $y = |x-2|$, $x = 1$, $x = 3$ and the x- axis is

(a) 1 sq. unit (b) 2 sq. units

(c) 3 sq. units (d) 4 sq. units

12. Area of the region bounded by the curve $y = x^2$ and the line $y = 4$ is

(a) $\frac{11}{3}$ sq. units (b) $\frac{32}{3}$ sq. units

(c) $\frac{43}{3}$ sq. units (d) $\frac{47}{3}$ sq. units

13. Area lying between the parabola $y^2 = 4x$ and its latus rectum is

(a) $\frac{1}{3}$ sq. units (b) $\frac{2}{3}$ sq. units

(c) $\frac{5}{3}$ sq. units (d) $\frac{8}{3}$ sq. units

14. The area bounded by the curve $y^2 = x$, line $y = 4$ and y-axis is

(a) $\frac{16}{3}$ sq. units (b) $\frac{64}{3}$ sq. units

(c) $7\sqrt{2}$ sq. units (d) none of these

15. The area bounded by the curve $x = 3y^2 - 9$ and the line $x = 0$, $y = 0$ and $y = 1$ is

(a) 8 sq. units (b) 8/3 sq. units

(c) 3/8 sq. unit (d) 3 sq. units

16. Find the area above x-axis, bounded by the curves $y = 2^{kx}$, $x = 0$ and $x = 2$.

(a) $\frac{4^k - 1}{k\log_e 2}$ (b) $\frac{2^k - 1}{2\log_e 2}$

(c) $\frac{3-k}{k\log_e 2}$ (d) $\frac{-1+3^k}{2\log_e 2}$

17. Find the area enclosed by the parabola $y^2 = x$ and the line $y + x = 2$ and the x-axis.

(a) $\frac{5}{6}$ sq. units (b) $\frac{7}{6}$ sq. units

(c) $\frac{6}{7}$ sq. units (d) $\frac{4}{7}$ sq. units

18. The area bounded by the curve $x^2 + y^2 = 1$ in first quadrant is

(a) $\frac{\pi}{4}$ sq. units (b) $\frac{\pi}{2}$ sq. units

(c) $\frac{\pi}{3}$ sq. units (d) $\frac{\pi}{6}$ sq. units

19. Area bounded by the curve $y = \cos x$ between $x = 0$ and $x = \frac{3\pi}{2}$ is

(a) 1 sq. unit (b) 2 sq. units

(c) 3 sq. units (d) 4 sq. units

20. Area of the region bounded by the curve $y = \tan x$, line $x = \frac{\pi}{4}$ and the x-axis is

(a) log 2 sq. units (b) $\frac{1}{2}\log 2$ sq. units

(c) $\frac{1}{3}\log 2$ sq. units (d) 5 log 2 sq. units

21. The area bounded by the curve $y = \sec^2 x$, $y = 0$ and $|x| = \frac{\pi}{3}$ is

(a) $\sqrt{3}$ sq. units (b) $\sqrt{2}$ sq. units

(c) $2\sqrt{3}$ sq. units (d) none of these

22. The area bounded by the curve $x^2 = 4y + 4$ and line $3x + 4y = 0$ is

(a) $\frac{25}{4}$ sq. units (b) $\frac{125}{8}$ sq. units

(c) $\frac{125}{16}$ sq. units (d) $\frac{125}{24}$ sq. units

23. The area bounded by the x-axis, the curve $y = f(x)$ and the lines $x = 1$, $x = b$ is equal to $\sqrt{b^2+1} - \sqrt{2}$ for all $b > 1$, then $f(x)$ is

(a) $\sqrt{x-1}$ (b) $\sqrt{x+1}$

(c) $\sqrt{x^2+1}$ (d) $x / \sqrt{x^2+1}$

24. The area (in sq. units) enclosed between the graph of $y = x^3$ and the lines $x = 0$, $y = 1$, $y = 8$ is

(a) $\frac{45}{4}$ (b) 14

(c) 7 (d) none of these

25. The area of the region bounded by the curve $y = \sqrt{16 - x^2}$ and x-axis is

(a) 8π sq. units (b) 20π sq. units

(c) 16π sq. units (d) 256π sq. units

26. Area of the region bounded by the curve $y = \cos x$ between $x = 0$ and $x = \pi$ is

(a) 2 sq. units (b) 4 sq. units

(c) 3 sq. units (d) 1 sq. unit

27. The area of the region bounded by parabola $y^2 = x$ and the straight line $2y = x$ is

(a) $\frac{4}{3}$ sq. units (b) 1 sq. unit

(c) $\frac{2}{3}$ sq. unit (d) $\frac{1}{3}$ sq. unit

28. The area of the region bounded by the curve $y = \sin x$ between the ordinates $x = 0$, $x = \frac{\pi}{2}$ and the x-axis is

(a) 2 sq. units (b) 4 sq. units

(c) 3 sq. units (d) 1 sq. unit

29. The area of the region bounded by the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ is

(a) 20π sq. units (b) $20\pi^2$ sq. units

(c) $16\pi^2$ sq. units (d) 25π sq. units

30. The area of the region bounded by the circle $x^2 + y^2 = 1$ is

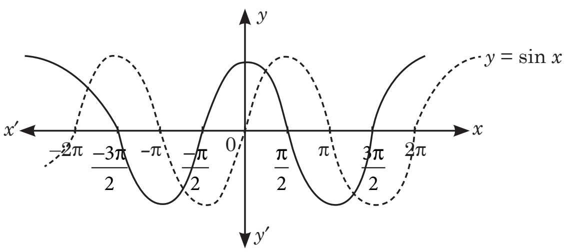
(a) 2π sq. units (b) π sq. units

(c) 3π sq. units (d) 4π sq. units

Case Based MCQs

Case I : Read the following passage and answer the questions from 31 to 35.

In a classroom, teacher explains the properties of a particular curve by saying that this particular curve has beautiful up and downs. It starts at 1 and heads down until π radian, and then heads up again and closely related to sine function and both follow each other, exactly $\frac{\pi}{2}$ radians apart as shown in figure.



31. Name the curve, about which teacher explained in the classroom.
(a) cosine (b) sine
(c) tangent (d) cotangent

32. Area of curve explained in the passage from 0 to $\frac{\pi}{2}$ is
(a) $\frac{1}{3}$ sq. unit (b) $\frac{1}{2}$ sq. unit
(c) 1 sq. unit (d) 2 sq. units

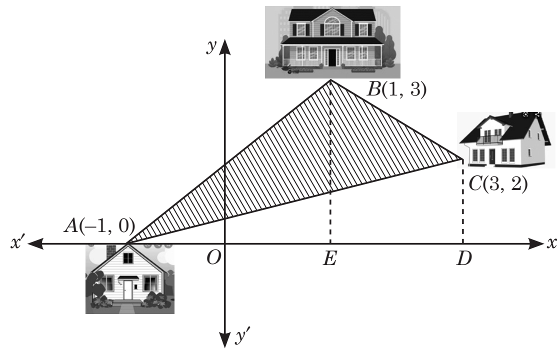
33. Area of curve discussed in classroom from $\frac{\pi}{2}$ to $\frac{3\pi}{2}$ is
(a) –2 sq. units (b) 2 sq. units
(c) 3 sq. units (d) –3 sq. units

34. Area of curve discussed in classroom from $\frac{3\pi}{2}$ to 2π is
(a) 1 sq. unit (b) 2 sq. units
(c) 3 sq. units (d) 4 sq. units

35. Area of explained curve from 0 to 2π is
(a) 1 sq. unit (b) 2 sq. units
(c) 3 sq. units (d) 4 sq. units

Case II : Read the following passage and answer the questions from 36 to 40.

Location of three houses of a society is represented by the points $A(-1, 0)$, $B(1, 3)$ and $C(3, 2)$ as shown in figure.



36. Equation of line AB is
(a) $y = \frac{3}{2}(x+1)$ (b) $y = \frac{3}{2}(x-1)$
(c) $y = \frac{1}{2}(x+1)$ (d) $y = \frac{1}{2}(x-1)$

37. Equation of line BC is
(a) $y = \frac{1}{2}x - \frac{7}{2}$ (b) $y = \frac{3}{2}x - \frac{7}{2}$
(c) $y = \frac{-1}{2}x + \frac{7}{2}$ (d) $y = \frac{3}{2}x + \frac{7}{2}$

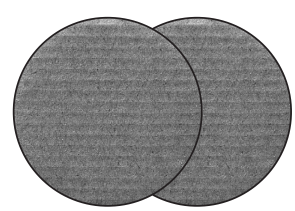
38. Area of region $ABCD$ is
(a) 2 sq. units (b) 4 sq. units
(c) 6 sq. units (d) 8 sq. units

39. Area of ΔADC is
(a) 4 sq. units (b) 8 sq. units
(c) 16 sq. units (d) 32 sq. units

40. Area of ΔABC is
(a) 3 sq. units (b) 4 sq. units
(c) 5 sq. units (d) 6 sq. units

Case III : Read the following passage and answer the questions from 41 to 45.

Ajay cut two circular pieces of cardboard and placed one upon other as shown in figure. One of the circle represents the equation $(x - 1)^2 + y^2 = 1$, while other circle represents the equation $x^2 + y^2 = 1$.

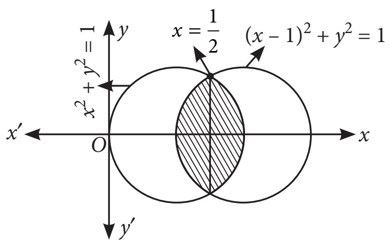


41. Both the circular pieces of cardboard meet each other at

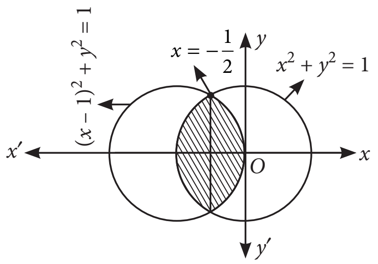
(a) $x = 1$ (b) $x = \frac{1}{2}$ (c) $x = \frac{1}{3}$ (d) $x = \frac{1}{4}$

42. Graph of given two curves can be drawn as

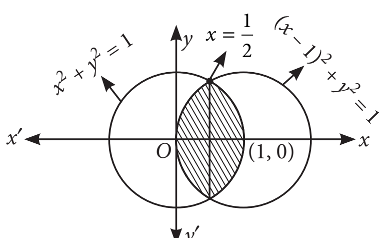
(a)



(b)



(c)



(d) None of these

43. Value of $\int_0^{1/2} \sqrt{1-(x-1)^2}\, dx$ is

(a) $\frac{\pi}{6} - \frac{\sqrt{3}}{8}$ (b) $\frac{\pi}{6} + \frac{\sqrt{3}}{8}$

(c) $\frac{\pi}{2} + \frac{\sqrt{3}}{4}$ (d) $\frac{\pi}{2} - \frac{\sqrt{3}}{4}$

44. Value of $\int_{1/2}^{1} \sqrt{1-x^2}\, dx$ is

(a) $\frac{\pi}{2} + \frac{\sqrt{3}}{4}$ (b) $\frac{\pi}{6} + \frac{\sqrt{3}}{8}$

(c) $\frac{\pi}{6} - \frac{\sqrt{3}}{8}$ (d) $\frac{\pi}{2} - \frac{\sqrt{3}}{4}$

45. Area of hidden portion of lower circle is

(a) $\left(\frac{2\pi}{3} + \frac{\sqrt{3}}{2}\right)$ sq. units

(b) $\left(\frac{\pi}{3} - \frac{\sqrt{3}}{8}\right)$ sq. units

(c) $\left(\frac{\pi}{3} + \frac{\sqrt{3}}{8}\right)$ sq. units

(d) $\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$ sq. units

Assertion & Reasoning Based MCQs

Directions (Q. 46-50) : In these questions, a statement of Assertion is followed by a statement of Reason is given. Choose the correct answer out of the following choices :

(a) Assertion and Reason both are correct statements and Reason is the correct explanation of Assertion.
(b) Assertion and Reason both are correct statements but Reason is not the correct explanation of Assertion.
(c) Assertion is correct statement but Reason is wrong statement.
(d) Assertion is wrong statement but Reason is correct statement.

46. Assertion : The area of the region bounded by the curve $y^2 = 4x$ and the line $x = 3$ is $8\sqrt{3}$ sq. units.

Reason : The area of the region bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$ is $\frac{9}{8}$ sq. units.

47. Assertion : The area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$ is $\frac{3}{2}(\pi - 2)$ sq. units.

Reason : Formula to calculate the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the line $\frac{x}{a} + \frac{y}{b} = 1$ is $\frac{ab}{4}(\pi - 2)$ sq. units.

48. Assertion : The area bounded by the parabola $y^2 = 4ax$ and the line $x = a$ and $x = 4a$ is $\frac{56a^2}{3}$ sq. units.

Reason : The area bounded by the curves $y = 3x$ and $y = x^2$ is 9.5 sq. units.

49. Assertion : The area bounded by the curves $y^2 = 4a^2(x-1)$ and lines $x = 1$ and $y = 4a$ is $\frac{8a}{3}$ sq. units.

Reason : The area enclosed between the parabola $y = x^2 - x + 2$ and the line $y = x + 2$ is $\frac{4}{3}$ sq. units.

50. Assertion : The area bounded by the curve $y = 2\cos x$ and the x-axis from $x = 0$ to $x = 2\pi$ is 8 sq. units.

Reason : The area bounded by the curve $y = \sin x$ between $x = \pi$ and $x = 2\pi$ is 4 sq. units.

SUBJECTIVE TYPE QUESTIONS

Very Short Answer Type Questions (VSA)

1. Find the area between the curve $y = 4 + 3x - x^2$ and x-axis.

2. Find the area of the ellipse $\frac{x^2}{4^2} + \frac{y^2}{9^2} = 1$.

3. Find the area of the region bounded by $y = |x|$, $x \le 5$ in the first quadrant.

4. Find the area of the smaller region bounded by $x^2 + y^2 = 9$ and the line $x = 1$.

5. Find the area of the region bounded by the curve $y = x + 1$ and the lines $x = 2$ and $x = 3$.

6. Find the area of the region bounded by the curve $x = 2y + 3$ and the lines $y = 1$ and $y = -1$.

7. Find the area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the lines $x = 0$ and $x = 2$.

8. Using integration, find the area of the region enclosed by the curves $y = \log x$, x-axis and ordinates $x = 1$, $x = 2$.

9. Find the area bounded by the curves $y = \sin x$, the line $x = 0$ and the line $x = 2\pi$.

10. Find the area bounded by the curve $y^2 = 9x$ and the lines $x = 1$, $x = 4$ and $y = 0$ in the first quadrant.

Short Answer Type Questions (SA-I)

11. Find the area bounded by the lines $y = ||x| - 1|$ and the x-axis.

12. Find the area of the region bounded by the curve $y^2 = x$ and the lines $x = 1$, $x = 4$ and the x-axis.

13. Find the area of the region bounded by $y^2 = 9x$, $x = 2$, $x = 4$ and the x-axis in the first quadrant.

14. If $y = 2\sin x + \sin 2x$ for $0 \le x \le 2\pi$, then find the area enclosed by the curve and x-axis.

15. Find the area of triangle whose two vertices formed from the x-axis and line $y = 3 - |x|$.

16. Find the area bounded by the curve $y = x|x|$, x-axis and the lines $x = -3$ and $x = 3$.

17. Find the area of region bounded by the curve $y^2 = 4x$ and the lines $x = 2$, $x = 4$ and the x-axis.

18. Find the area of the region bounded by the curve $y = \sqrt{4 - x^2}$ and x-axis.

19. Using integration, find the area of region bounded between the line $x = 2$ and the parabola $y^2 = 8x$.

20. Draw the region lying in first quadrant and bounded by $y = 9x^2$, $x = 0$, $y = 1$ and $y = 4$. Also, find the area of region using integration.

Short Answer Type Questions (SA-II)

21. Find the area of region bounded by $y = \sqrt{x}$ and $y = x$.

22. Draw the graph of curve $y = |x + 1|$.

Hence, evaluate $\int_{-4}^{2} |x+1|\, dx$.

23. Find the area of the region bounded by the curve $y=\sqrt{1-x^2}$, line $y=x$ and the positive x-axis.

24. Find the area of the region in the first quadrant enclosed by the x-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$.

25. Find the area bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the ordinates $x = ae$ and $x = 0$, where $b^2 = a^2(1 - e^2)$ and $e < 1$.

26. Find the area of the region bounded by the parabola $y^2 = 2x + 1$ and the line $x - y - 1 = 0$.

27. Find the area of smaller region bounded by the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ and the line $\frac{x}{4}+\frac{y}{3}=1$.

28. Find the area bounded by the curve $y = 2x - x^2$ and the straight line $y = -x$.

29. AOB is a positive quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where $OA = a$, $OB = b$. Find the area between the arc AB and chord AB of the ellipse.

30. Find the area of the triangle formed by the tangent and normal at the point $(1, \sqrt{3})$ on the circle $x^2 + y^2 = 4$ and the x-axis.

31. Draw the region bounded by $y = 2x - x^2$ and x-axis and find its area using integration.

32. Determine the area under the curve $y=\sqrt{a^2-x^2}$ included between the lines $x = 0$ and $x = a$.

33. Find the area of the region bounded by $y = |x - 1|$ and $y = 1$.

34. If the area bounded the curve $y^2 = 16x$ and line $y = mx$ is $\frac{2}{3}$, then find the value of m.

35. Find the area enclosed between the parabola $4y = 3x^2$ and the straight line $3x - 2y + 12 = 0$.

Long Answer Type Questions (LA)

36. Find the area bounded by lines $y = 4x + 5$, $y = 5 - x$ and $4y = x + 5$.

37. Using integration, find the area of the region bounded by the curves :

$y = |x + 1| + 1$, $x = -3$, $x = 3$ and $y = 0$.

38. Using integration, find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.

39. Find the area of the region bounded by the parabola $y = x^2$ and $y = |x|$.

40. Find the area bounded by the circle $x^2 + y^2 = 16$ and the line $\sqrt{3}y = x$ in the first quadrant, using integration.

ANSWERS

OBJECTIVE TYPE QUESTIONS

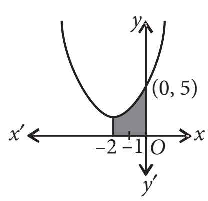
1. **(b) :** We have, $y = x^2 + 4x + 5 = (x + 2)^2 + 1$

$\therefore$ Required area $=\int_{-2}^{0}(x^2+4x+5)\,dx$

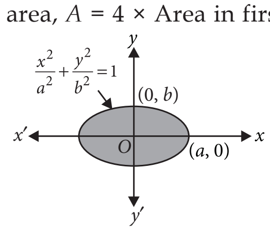
$=\left[\frac{x^3}{3}+2x^2+5x\right]_{-2}^{0}$

$=-\left[-\frac{8}{3}+8-10\right]$

$=2+\frac{8}{3}=\frac{14}{3}=4\frac{2}{3}$ sq. units



2. **(c) :** Total area, $A = 4 \times$ Area in first quadrant



$= 4\times\int_0^a y\,dx = 4\int_0^a \frac{b}{a}\sqrt{a^2-x^2}\,dx$

$=\frac{4b}{a}\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a=\pi ab$ sq. units

3. **(b) :** We have, $2x^2 + y^2 = 2$

$\Rightarrow \frac{x^2}{1}+\frac{y^2}{2}=1$, which is an ellipse

Here, $a = 1$ and $b = \sqrt{2}$

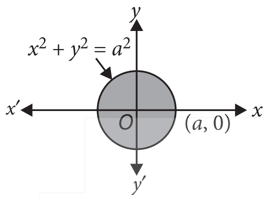
$\because$ Area bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is πab

$\therefore$ Required area = $\pi\sqrt{2}$ sq. units.

4. (b) : We have, $x^2 + y^2 = a^2$, which is a circle with centre (0, 0) and radius a.

$\therefore$ Required area = 4 × Area in the first quadrant

$$= 4\int_0^a \sqrt{a^2-x^2}\,dx$$



$$= 4\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$$

$$= 4\left(\frac{a^2}{2}\right)\frac{\pi}{2} = \pi a^2 \text{ sq. units}$$

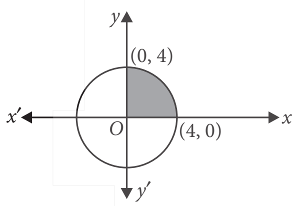
5. (a) : Here $a^2 = 4$ and $b^2 = 9$.

Since, area of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ is πab sq. units.

$\therefore$ Required area = $\pi \times 2 \times 3 = 6\pi$ sq. units.

6. (a) : Given curve is a circle with centre (0, 0) and radius 4.

$\therefore$ Required area



$$= \int_0^4 \sqrt{16-x^2}\,dx$$

$$= \left[\frac{x}{2}\sqrt{16-x^2}+\frac{16}{2}\sin^{-1}\frac{x}{4}\right]_0^4 = 4\pi \text{ sq. units}$$

7. (b) : We have $\frac{x^2}{25}+\frac{y^2}{9}=1$, which is an ellipse

Here, $a = 5$ and $b = 3$

Since, area of region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is πab.

$\therefore$ Required area = π (5) (3) = 15π sq. units

8. (c) : Given, $\int_1^b f(x)dx = (b-1)\sin(3b+4)$

Area function $= \int_1^x f(x)dx = (x-1)\sin(3x+4)$

On differentiating, we get

$f(x) = \sin(3x + 4) + 3(x - 1)\cdot \cos(3x + 4)$

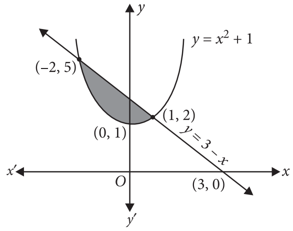
9. (d) : We have, $y = x^2 + 1$ ...(i)

and $x + y = 3$ ...(ii)

Solving (i) and (ii), we get

$x^2 + x - 2 = 0 \Rightarrow x = -2, 1$

$\therefore$ Required area



$$= \int_{-2}^{1} \{3-x-(x^2+1)\}\,dx$$

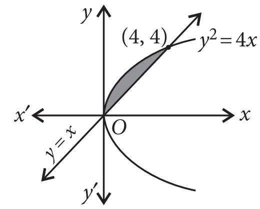
$$= \left[2x-\frac{x^2}{2}-\frac{x^3}{3}\right]_{-2}^{1}$$

$$= \left(2-\frac{1}{2}-\frac{1}{3}\right)-\left(-4-2+\frac{8}{3}\right)=\frac{9}{2} \text{ sq. units}$$

10. (a) : We have, $y^2 = 4x$ ...(i)

and $y = x$ ...(ii)

$\therefore$ Required area



$$= \int_0^4 (\sqrt{4x}-x)dx = \int_0^4 (2x^{1/2}-x)\,dx$$

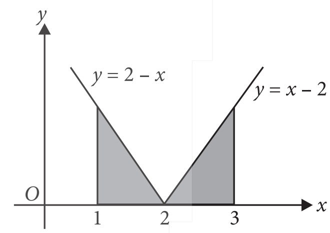
$$= \left[2\frac{x^{3/2}}{3/2}-\frac{x^2}{2}\right]_0^4 = \frac{4}{3}(4^{3/2})-\frac{4^2}{2}$$

$$= \frac{32}{3}-8=\frac{8}{3} \text{ sq. units}$$

11. (a) : We have, $y = -x + 2\ \forall\ x < 2$ ...(i)

$y = x - 2\ \forall\ x \ge 2$ ...(ii)

and $x = 1$, $x = 3$



$\therefore$ Required area

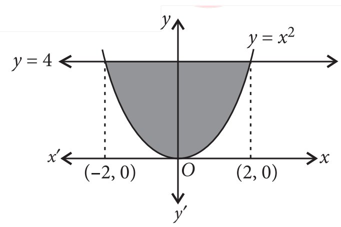
$$= \int_1^2 (2-x)\,dx + \int_2^3 (x-2)\,dx = \left[2x-\frac{x^2}{2}\right]_1^2+\left[\frac{x^2}{2}-2x\right]_2^3$$

$$= \left[(4-2)-\left(2-\frac{1}{2}\right)\right]+\left[\left(\frac{9}{2}-\frac{4}{2}\right)-(6-4)\right]$$

$$= \frac{1}{2}+\frac{1}{2}=1 \text{ sq. unit}$$

12. (b) : We have, $y = x^2$ ...(i)

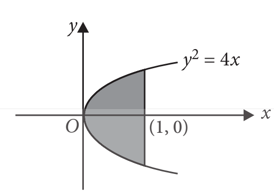
and $y = 4$ ...(ii)



$\therefore$ Required area

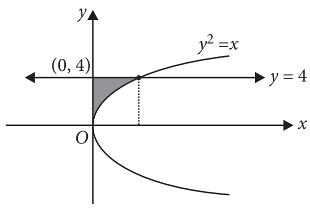
$$= 2\int_0^2 (4-x^2)dx = \left[2\left(4x-\frac{x^3}{3}\right)\right]_0^2 = \frac{32}{3} \text{ sq. units}$$

13. (d) : We know that the area of region bounded by the parabola $y^2 = 4ax$ and its latus rectum is $\frac{8}{3}a^2$ sq. units.



Here, $a = 1$, therefore required area $= \frac{8}{3}$ sq. units

14. (b) : We have, $y^2 = x$, which is a parabola with vertex (0, 0) and line $y = 4$

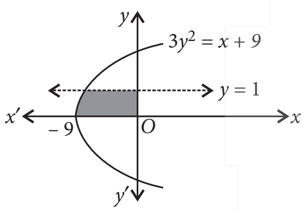


$\therefore$ Required area

$$= \int_0^4 y^2\, dy$$

$$= \left[\frac{y^3}{3}\right]_0^4 = \frac{64}{3} \text{ sq. units}$$

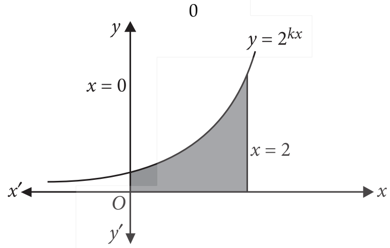
15. (a) : We have, $x = 3y^2 - 9 \Rightarrow 3y^2 = x + 9$



$\therefore$ Required area

$$= \left|\int_0^1 (3y^2 - 9)dy\right| = \left|[y^3 - 9y]_0^1\right| = |1 - 9| = 8 \text{ sq. units}$$

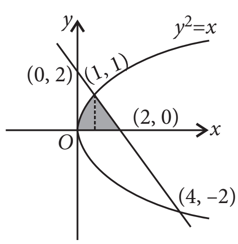
16. (a) : Required area $= \int_0^2 y\, dx$



$$= \int_0^2 2^{kx}\, dx = \left[\frac{2^{kx}}{k \log_e 2}\right]_0^2$$

$$= \frac{2^{2k}}{k\log_e 2} - \frac{1}{k\log_e 2} = \frac{4^k - 1}{k \log_e 2}$$

17. (b) : The given line and parabola meet at the points (1, 1) and (4, –2).



$\therefore$ Required area

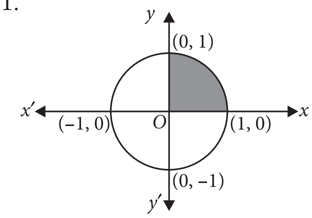
$$= \int_0^1 \sqrt{x}\, dx + \int_1^2 (2 - x)\, dx$$

$$= \left[\frac{x^{3/2}}{3/2}\right]_0^1 + \left[2x - \frac{x^2}{2}\right]_1^2$$

$$= \frac{2}{3}(1 - 0) + \left(2 \times 2 - \frac{2^2}{2}\right) - \left(2 - \frac{1}{2}\right)$$

$$= \frac{2}{3} + 2 - \frac{3}{2} = \frac{4 + 12 - 9}{6} = \frac{7}{6} \text{ sq. units}$$

18. (a) : We have, $x^2 + y^2 = 1$, which is a circle with centre (0, 0) and radius = 1.



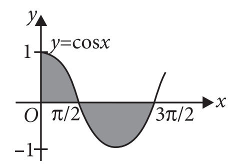
Required area

$$= \int_0^1 \sqrt{1 - x^2}\, dx$$

$$= \left[\frac{x}{2}\sqrt{1 - x^2} + \frac{1}{2}\sin^{-1}\frac{x}{1}\right]_0^1$$

$$= \left[\frac{1}{2}\sin^{-1} 1\right] = \left(\frac{1}{2} \times \frac{\pi}{2}\right) = \frac{\pi}{4} \text{ sq. units}$$

19. (c) : We have, $y = \cos x$, whose graph is shown below, between $x = 0$ and $x = \frac{3\pi}{2}$



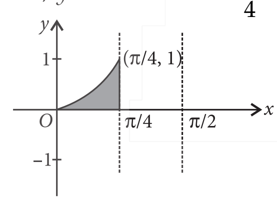
$\therefore$ Required area

$$= \int_0^{\pi/2} \cos x\, dx + \left|\int_{\pi/2}^{3\pi/2} \cos x\, dx\right|$$

$$= [\sin x]_0^{\pi/2} + \left|[\sin x]_{\pi/2}^{3\pi/2}\right|$$

$= 1 + |(-1 - 1)| = 1 + 2 = 3$ sq. units

20. (b) : We have, $y = \tan x$ and $x = \frac{\pi}{4}$

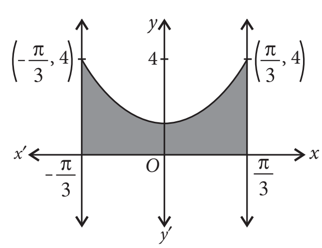


$\therefore$ Required area

$$= \int_0^{\pi/4} \tan x\, dx = [-\log|\cos x|]_0^{\pi/4} = -\log\frac{1}{\sqrt{2}} + \log 1$$

$$= \log\sqrt{2} = \frac{1}{2}\log 2 \text{ sq. units}$$

21. (c) : We have, $y = \sec^2 x$ and $y = 0$ and $x = \frac{\pi}{3}, -\frac{\pi}{3}$



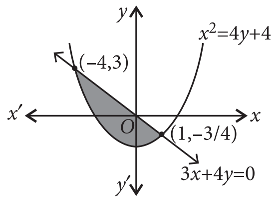
$\therefore$ Required area

$$= \int_{-\pi/3}^{\pi/3} \sec^2 x\, dx = [\tan x]_{-\pi/3}^{\pi/3} = 2\sqrt{3} \text{ sq. units}$$

22. (d) : We have, $x^2 = 4y + 4$ ...(i)

and $3x + 4y = 0$ ...(ii)

Solving (i) and (ii), we get $x = -4, 1$



∴ Required area

$= \int_{-4}^{1}\left(-\frac{3x}{4}-\frac{x^2}{4}+1\right)dx$

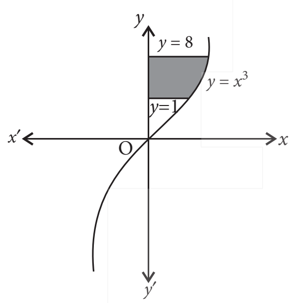
$= -\frac{3}{8}(1-16)-\frac{1}{12}(1+64)+5 = \frac{45}{8}-\frac{5}{12} = \frac{125}{24}$ sq. units

23. (d) : We have, $\int_1^b f(x)\,dx = \sqrt{b^2+1}-\sqrt{2}$

On differentiating w.r.t. b, we get

$f(b)=\frac{2b}{2\sqrt{b^2+1}} \Rightarrow f(x)=\frac{x}{\sqrt{x^2+1}}$

24. (a) : Given curve is $y = x^3$ or $x = y^{1/3}$



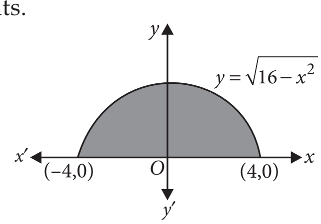
∴ Required area

$=\int_1^8 y^{1/3}\,dy=\left[\frac{y^{4/3}}{4/3}\right]_1^8=\frac{3}{4}\left[8^{4/3}-1^{4/3}\right]$

$=\frac{3}{4}\times(16-1) = \frac{3}{4}\times 15 = \frac{45}{4}$ sq. units

25. (a) : We have, $y=\sqrt{16-x^2} \Rightarrow y^2 = 16 - x^2$

$\Rightarrow x^2 + y^2 = 4^2$, which is a circle with centre (0, 0) and radius 4 units.

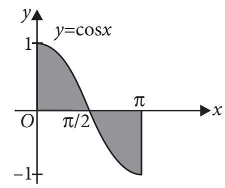


∴ Required area

$= \int_{-4}^{4}\sqrt{4^2-x^2}\,dx = \left[\frac{x}{2}\sqrt{4^2-x^2}+\frac{4^2}{2}\sin^{-1}\frac{x}{4}\right]_{-4}^{4}$

$=\left[8\sin^{-1}(1)-8\sin^{-1}(-1)\right] = \frac{8\pi}{2}+\frac{8\pi}{2}=8\pi$ sq. units

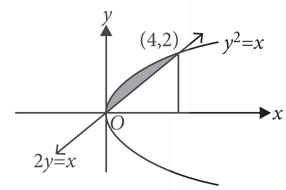
26. (a) : We have, $y = \cos x$



∴ Required area

$= 2\int_0^{\pi/2}\cos x\,dx = 2[\sin x]_0^{\pi/2} = 2$ sq. units

27. (a) : We have $2y = x$ ... (i), a straight line, and $y^2 = x$ ...(ii), a parabola with vertex (0, 0).
Solving (i) and (ii), we get $x = 0$ and $x = 4$.

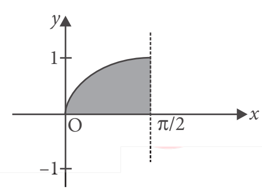


∴ Required area

$= \int_0^4\left(\sqrt{x}-\frac{x}{2}\right)dx = \left[\frac{2}{3}x^{3/2}-\frac{x^2}{4}\right]_0^4 = \frac{2}{3}\times 8-\frac{16}{4}$

$=\frac{16-12}{3}=\frac{4}{3}$ sq. units

28. (d) : We have, $y = \sin x,\ 0\le x\le\frac{\pi}{2}$



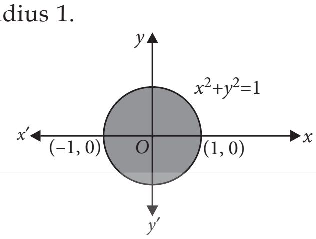
∴ Required area

$= \int_0^{\pi/2}\sin x\,dx = [-\cos x]_0^{\pi/2} = -[0-1] = 1$ sq. unit

29. (a) : Area of the region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is πab sq. units

∴ Required area = $\pi \times 5 \times 4 = 20\pi$ sq. units

30. (b) : We have, $x^2 + y^2 = 1$, a circle with centre (0, 0) and radius 1.



$\therefore$ Required area

$$= 4\int_0^1 \sqrt{1-x^2}\,dx = 4\left[\frac{x}{2}\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}x\right]_0^1$$

$$= 4\times\frac{1}{2}\times\frac{\pi}{2} = \pi \text{ sq. units}$$

31. (a) : Here, teacher explained about cosine curve.

32. (c) : Required area = $\int_0^{\pi/2} \cos x\,dx$

$= [\sin x]_0^{\pi/2} = \sin\frac{\pi}{2} - \sin 0 = 1 - 0 = 1$ sq. unit

33. (b) : Required area = $\left|\int_{\pi/2}^{3\pi/2} \cos x\,dx\right| = \left|[\sin x]_{\pi/2}^{3\pi/2}\right|$

$= \left|\sin\frac{3\pi}{2} - \sin\frac{\pi}{2}\right| = |-1-1| = |-2|$

= 2 sq. units [Since, area can't be negative]

34. (a) : Required area = $\int_{3\pi/2}^{2\pi} \cos x\,dx = [\sin x]_{3\pi/2}^{2\pi}$

$= \sin 2\pi - \sin\frac{3\pi}{2} = 0 - (-1) = 1$ sq. unit

35. (d) : Required area

$$= \int_0^{\pi/2}\cos x\,dx + \left|\int_{\pi/2}^{3\pi/2}\cos x\,dx\right| + \int_{3\pi/2}^{2\pi}\cos x\,dx$$

= 1 + 2 + 1 = 4 sq. units

36. (a) : Equation of line AB is

$y - 0 = \frac{3-0}{1+1}(x+1) \Rightarrow y = \frac{3}{2}(x+1)$

37. (c) : Equation of line BC is

$y - 3 = \frac{2-3}{3-1}(x-1)$

$\Rightarrow \quad y = -\frac{1}{2}x + \frac{1}{2} + 3 \Rightarrow y = \frac{-1}{2}x + \frac{7}{2}$

38. (d) : Area of region $ABCD$

= Area of ΔABE + Area of region $BCDE$

$$= \int_{-1}^{1}\frac{3}{2}(x+1)\,dx + \int_1^3\left(\frac{-1}{2}x+\frac{7}{2}\right)dx$$

$$= \frac{3}{2}\left[\frac{x^2}{2}+x\right]_{-1}^{1} + \left[\frac{-x^2}{4}+\frac{7}{2}x\right]_1^3$$

$$= \frac{3}{2}\left[\frac{1}{2}+1-\frac{1}{2}+1\right] + \left[\frac{-9}{4}+\frac{21}{2}+\frac{1}{4}-\frac{7}{2}\right]$$

= 3 + 5 = 8 sq. units

39. (a) : Equation of line AC is $y - 0 = \frac{2-0}{3+1}(x+1)$

$\Rightarrow \quad y = \frac{1}{2}(x+1)$

$\therefore$ Area of $\Delta ADC = \int_{-1}^{3}\frac{1}{2}(x+1)\,dx = \left[\frac{x^2}{4}+\frac{1}{2}x\right]_{-1}^{3}$

$= \frac{9}{4}+\frac{3}{2}-\frac{1}{4}+\frac{1}{2} = 4$ sq. units

40. (b) : Area of ΔABC = Area of region $ABCD$ – Area of ΔACD = 8 – 4 = 4 sq. units

41. (b) : We have, $(x-1)^2 + y^2 = 1$

$\Rightarrow \quad y = \sqrt{1-(x-1)^2}$ ...(i)

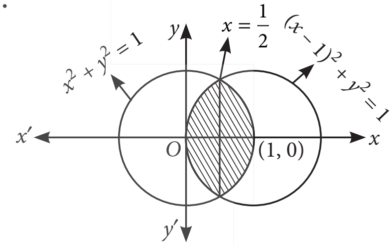
Also, $x^2 + y^2 = 1 \Rightarrow y = \sqrt{1-x^2}$ ...(ii)

From (i) and (ii), we get

$\sqrt{1-(x-1)^2} = \sqrt{1-x^2}$

$\Rightarrow \quad (x-1)^2 = x^2 \Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$

42. (c) :



43. (a) : $\int_0^{1/2}\sqrt{1-(x-1)^2}\,dx$

$$= \left[\frac{x-1}{2}\sqrt{1-(x-1)^2} + \frac{1}{2}\sin^{-1}\left(\frac{x-1}{1}\right)\right]_0^{1/2}$$

$$= \frac{1}{2}\left(\frac{1}{2}-1\right)\sqrt{1-\frac{1}{4}} + \frac{1}{2}\sin^{-1}\left(-\frac{1}{2}\right) - \left(-\frac{1}{2}\right)(0) - \frac{1}{2}\sin^{-1}(-1)$$

$$= \left[\frac{-1}{4}\cdot\frac{\sqrt{3}}{2} - \frac{1}{2}\cdot\frac{\pi}{6} + 0 + \frac{1}{2}\cdot\frac{\pi}{2}\right] = \frac{-\sqrt{3}}{8} - \frac{\pi}{12} + \frac{\pi}{4}$$

$$= \frac{\pi}{6} - \frac{\sqrt{3}}{8}$$

44. (c) : $\int_{1/2}^{1}\sqrt{1-x^2}\,dx = \left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x\right]_{1/2}^{1}$

$$= 0 + \frac{1}{2}\sin^{-1}(1) - \frac{1}{4}\sqrt{1-\frac{1}{4}} - \frac{1}{2}\sin^{-1}\left(\frac{1}{2}\right)$$

$$= \frac{\pi}{4} - \frac{\sqrt{3}}{8} - \frac{\pi}{12} = \frac{\pi}{6} - \frac{\sqrt{3}}{8}$$

45. (d) : Required area

$$= 2\left[\int_0^{1/2}\sqrt{1-(x-1)^2}\,dx + \int_{1/2}^{1}\sqrt{1-x^2}\,dx\right]$$

$$= 2\left[\frac{\pi}{6} - \frac{\sqrt{3}}{8} + \frac{\pi}{6} - \frac{\sqrt{3}}{8}\right]$$

$$= 2\left[\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right] = \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)\text{sq. units}$$

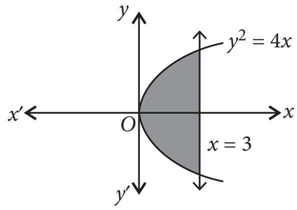
46. (b) : Assertion : We have, $y^2 = 4x$ and $x = 3$.

$\therefore$ Required area

$$= 2\int_0^3 |y|\,dx = 2\int_0^3 2\sqrt{x}\,dx$$

$$= 4\left[\frac{x^{3/2}}{3/2}\right]_0^3$$

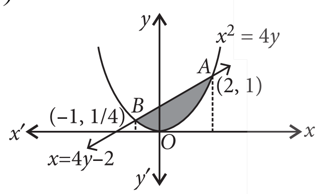
$$= \frac{8}{3}(3\sqrt{3}) = 8\sqrt{3} \text{ sq. units}$$



Reason : We have, $x^2 = 4y \Rightarrow y = \frac{x^2}{4}$

and $x = 4y - 2 \Rightarrow y = \frac{x+2}{4}$.

The point of intersection of given curves are $A(2, 1)$ and $B\left(-1, \frac{1}{4}\right)$.



$\therefore$ Required area $= \int_{-1}^{2}\left(\frac{x+2}{4}\right)dx - \int_{-1}^{2}\frac{x^2}{4}dx$

$$= \frac{1}{4}\left[\frac{x^2}{2} + 2x\right]_{-1}^{2} - \frac{1}{4}\left[\frac{x^3}{3}\right]_{-1}^{2}$$

$$= \frac{1}{4}\left(6 + \frac{3}{2}\right) - \frac{1}{12} \times 9 = \frac{15}{8} - \frac{3}{4} = \frac{9}{8} \text{ sq. units}$$

47. (a) : Clearly, reason is correct statement.

Now, we have, equation of ellipse

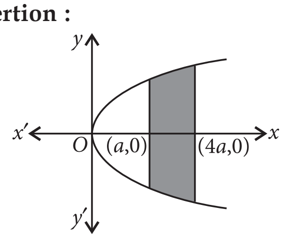
$\frac{x^2}{9} + \frac{y^2}{4} = 1$ and line $\frac{x}{3} + \frac{y}{2} = 1$

$\therefore$ Here, $a = 3$, $b = 2$

$\therefore$ Required area $= \frac{ab}{4}(\pi - 2)$

$$= \frac{3 \times 2}{4}(\pi - 2) = \frac{3}{2}(\pi - 2) \text{ sq. units}$$

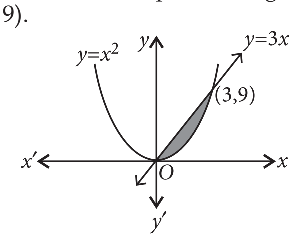
48. (c) : Assertion :



Required area $= 2\int_a^{4a}\sqrt{4ax}\,dx = 4\sqrt{a}\left[\frac{x^{3/2}}{3/2}\right]_a^{4a}$

$$= \frac{8}{3}\sqrt{a}\left(8a^{3/2} - a^{3/2}\right) = \frac{56a^2}{3} \text{ sq. units}$$

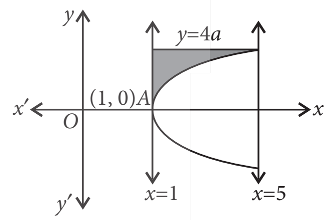
Reason : The intersection points of given curves are (0, 0) and (3, 9).



$\therefore$ Required area $= \int_0^3 (3x - x^2)\,dx$

$$= \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^3 = \frac{27}{6} = 4.5 \text{ sq. units}$$

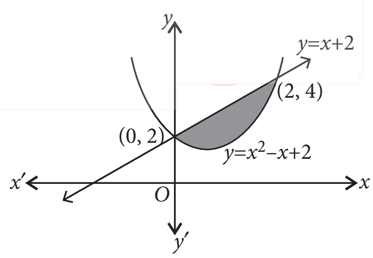
49. (d) : Assertion : On solving $y^2 = 4a^2(x - 1)$ and $y = 4a$, we get $x = 5$



$\therefore$ Required area $= \int_1^5 (4a - 2a\sqrt{x-1})\,dx$

$$= \left[4ax - 2a \cdot \frac{(x-1)^{3/2}}{3/2}\right]_1^5 = \frac{16a}{3} \text{ sq. units}$$

Reason : Given, parabola $y = x^2 - x + 2$ and the line $y = x + 2$ intersects each other at points (0, 2) and (2, 4).

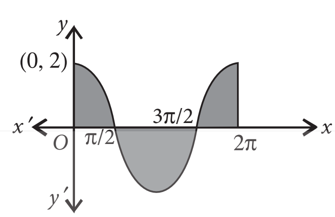


$\therefore$ Required area $= \int_0^2 \left[(x+2) - (x^2 - x + 2)\right] dx$

$$= \int_0^2 (-x^2 + 2x)\,dx = \left[\frac{-x^3}{3} + x^2\right]_0^2 = -\frac{8}{3} + 4 = \frac{4}{3} \text{ sq. units}$$

50. (c) : Assertion : We have, $y = 2\cos x$

Let us draw the graph of $2\cos x$ between 0 to 2π.



$\therefore$ Required area

$$= \int_0^{\pi/2} 2\cos x\, dx + \left|\int_{\pi/2}^{3\pi/2} 2\cos x\, dx\right| + \int_{3\pi/2}^{2\pi} 2\cos x\, dx$$

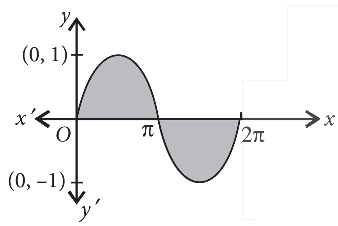
$$= 2[\sin x]_0^{\pi/2} + \left|[2\sin x]_{\pi/2}^{3\pi/2}\right| + [2\sin x]_{3\pi/2}^{2\pi}$$

$$= 2\left[\sin\frac{\pi}{2} - 0\right] + \left|2\left[\sin\frac{3\pi}{2} - \sin\frac{\pi}{2}\right]\right| + 2\left[\sin 2\pi - \sin\frac{3\pi}{2}\right]$$

= 2 + 2 × 2 + 2 = 2 + 4 + 2 = 8 sq. units

Reason : We have $y = \sin x$

Let us draw a graph of $\sin x$.



$\therefore$ Required area

$$= \left|\int_{\pi}^{2\pi} \sin x\, dx\right| = \left|[-\cos x]_{\pi}^{2\pi}\right|$$

$= |-\cos 2\pi + \cos\pi|$

$= |-1 - 1| = 2$ sq. units

SUBJECTIVE TYPE QUESTIONS

1. We have, $y = 4 + 3x - x^2$, a parabola with vertex at $\left(\frac{3}{2}, \frac{25}{4}\right)$.

Putting $y = 0$, we get $x^2 - 3x - 4 = 0$

$\Rightarrow (x - 4)(x + 1) = 0 \Rightarrow x = -1$ or $x = 4$

$\therefore$ Required area $= \int_{-1}^{4} (4 + 3x - x^2)\, dx$

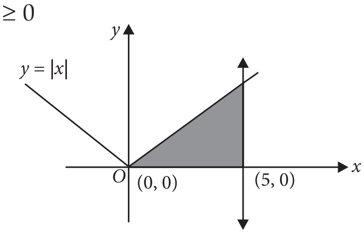
$$= \left[4x + \frac{3x^2}{2} - \frac{x^3}{3}\right]_{-1}^{4} = \frac{125}{6} \text{ sq. units}$$

2. Since, area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab.

$\therefore$ Required area = π × 4 × 9 = 36π sq. units.

3. We have, $y = -x$, if $x < 0$ ... (i)

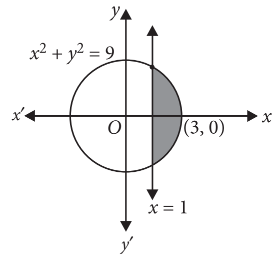
$y = x$, if $x \geq 0$ ...(ii)



$\therefore$ Required area

$$= \int_0^5 x\, dx = \left[\frac{x^2}{2}\right]_0^5 = \frac{25}{2} \text{ sq. units}$$

4. We have, $x^2 + y^2 = 9$ and $x = 1$.



$\therefore$ Required area

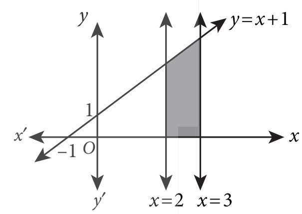
$$= 2\left[\int_1^3 \sqrt{9 - x^2}\, dx\right]$$

$$= 2\left[\frac{x}{2}\sqrt{9 - x^2} + \frac{9}{2}\sin^{-1}\frac{x}{3}\right]_1^3$$

$$= 2\left[\frac{9}{2}\sin^{-1}\frac{3}{3} - \frac{1}{2}\sqrt{8} - \frac{9}{2}\sin^{-1}\frac{1}{3}\right]$$

$$= -\sqrt{8} + 9\left(\sin^{-1}1 - \sin^{-1}\frac{1}{3}\right)$$

5. We have, $y = x + 1$, which is a straight line

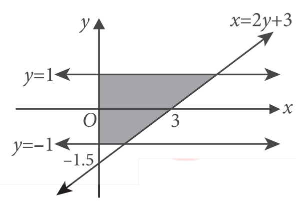


$\therefore$ Required area

$$= \int_2^3 (x + 1)\, dx = \left[\frac{x^2}{2} + x\right]_2^3 = \left(\frac{9}{2} + 3\right) - \left(\frac{4}{2} + 2\right)$$

$$= \frac{15}{2} - 4 = \frac{7}{2} \text{ sq. units}$$

6. We have $x = 2y + 3$, a straight line

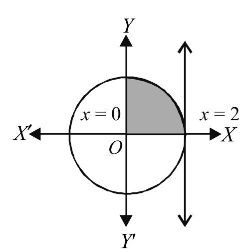


$\therefore$ Required area

$$= \int_{-1}^{1} (2y + 3)\, dy = \left[y^2 + 3y\right]_{-1}^{1}$$

= (1 + 3) – (1 – 3) = 4 + 2 = 6 sq. units

7. Required area $= \int_0^2 y\, dx$



$$= \int_0^2 \sqrt{4 - x^2}\, dx$$

$$= \left[\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2$$

$= [0 + 2\sin^{-1}(1)] - [0 - 0]$

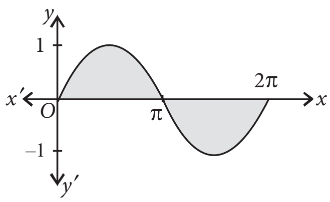
$$= 2\frac{\pi}{2} = \pi \text{ sq. units}$$

8. Required area $= \int_1^2 \log x\, dx = [x\log x - 1]_1^2$

$= 2\log 2 - 1 = \log 4 - \log e$

$= \log\left(\frac{4}{e}\right)$ sq. units

9.



Required area

$= \int_0^{\pi} (\sin x)\,dx + \left| \int_{\pi}^{2\pi} \sin x \, dx \right| = [-\cos x]_0^{\pi} + [|-\cos x|]_{\pi}^{2\pi}$

$= -\cos\pi + \cos 0 + |-\cos 2\pi + \cos\pi|$

$= 1 + 1 + |-1 - 1| = 2 + |-2| = 2 + 2 = 4$ sq. units

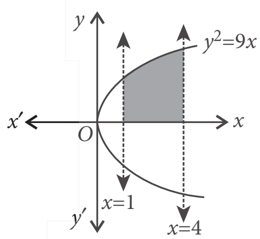
10. We have, $y^2 = 9x$ and lines $x = 1$, $x = 4$

$\therefore$ Required area

$= \int_1^4 3\sqrt{x}\,dx = 3\left[\frac{x^{3/2}}{3/2}\right]_1^4$

$= 2(4^{3/2} - 1) = 2(8 - 1)$

$= 14$ sq. units



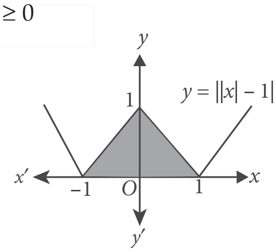
11. We have, $y = |x - 1|$, if $x \geq 0$

$= \begin{cases} (x-1), & \text{if } x \geq 1 \\ -(x-1), & \text{if } x < 1 \end{cases}$ and

$y = |-x-1| = |-(x+1)|$

$= |x+1|$ if $x < 0$

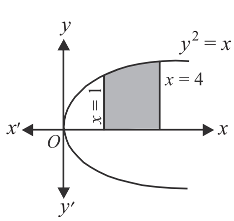
$= \begin{cases} (x+1), & \text{if } x \geq -1 \\ -(x+1), & \text{if } x < -1 \end{cases}$



$\therefore$ Required area $= 2\int_0^1 (1-x)\,dx$

$= 2\left[x - \frac{x^2}{2}\right]_0^1 = 2 \times \frac{1}{2} = 1$ sq. unit

12. Since the given parabola $y^2 = x$ is symmetrical about positive x-axis



$\therefore$ Required area $= \int_1^4 y\,dx$

$= \int_1^4 \sqrt{x}\,dx = \int_1^4 x^{1/2} dx = \left[\frac{x^{3/2}}{3/2}\right]_1^4$

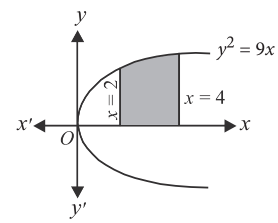
$= \frac{2}{3}[4^{3/2} - 1] = \frac{2}{3}[8 - 1] = \frac{14}{3}$ sq. units.

13. The given parabola is $y^2 = 9x$. It is symmetrical about positive x-axis.

$\therefore$ Required area $= \int_2^4 y\,dx$

$= \int_2^4 3\sqrt{x}\,dx$

$= 3\int_2^4 x^{1/2}\,dx = 3\left[\frac{x^{3/2}}{3/2}\right]_2^4$



$= 2\left[4^{3/2} - 2^{3/2}\right] = 2[8 - 2\sqrt{2}] = (16 - 4\sqrt{2})$ sq. units.

14. We have, $y = 2\sin x + \sin 2x$, $0 \leq x \leq 2\pi$

Putting $y = 0$, we get $2\sin x + \sin 2x = 0$

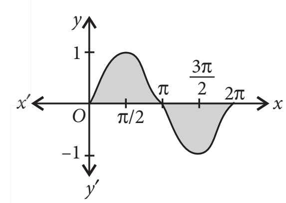
$\Rightarrow$ $2\sin x + 2\sin x \cos x = 0$

$\Rightarrow$ $2\sin x\,(1 + \cos x) = 0$

$\Rightarrow$ $\sin x = 0$ or $\cos x = -1$

$\Rightarrow$ $x = 0, \pi, 2\pi$

$\therefore$ Required area



$= 2\int_0^{\pi} (2\sin x + \sin 2x)dx$

$= 2\left[-2\cos x - \frac{\cos 2x}{2}\right]_0^{\pi}$

$= -2\left[\left(2\cos\pi + \frac{\cos 2\pi}{2}\right) - \left(2\cos 0 + \frac{\cos 0}{2}\right)\right]$

$= -2\left[\left(-2 + \frac{1}{2}\right) - \left(2 + \frac{1}{2}\right)\right] = -2\left[-2 + \frac{1}{2} - 2 - \frac{1}{2}\right]$

$= -2[-4] = 8$ sq. units

15. We have, $y = 3 - |x|$

$\Rightarrow$ $y = 3 + x$, $\forall\, x < 0$ ...(i)

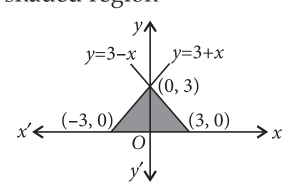
and $y = 3 - x$, $\forall\, x \geq 0$ ...(ii)

$\therefore$ Required area = area of shaded region

$= \left| 2\int_{-3}^{0} (3 + x)\,dx \right| = \left| 2\left[3x + \frac{x^2}{2}\right]_{-3}^{0} \right|$

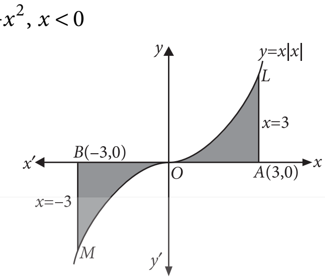
$= \left| -2\left[-9 + \frac{9}{2}\right] \right|$

$= \left| -2 \times \frac{-9}{2} \right| = 9$ sq. units



16. The equation of the curve is

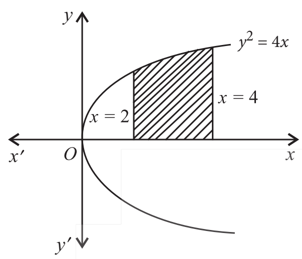
$y = x|x| = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$



$\therefore$ Required area = 2(Area of region shaded in first quadrant)

$$= 2\int_0^3 x^2\,dx = 2\times\left[\frac{x^3}{3}\right]_0^3 = 2\times 9 = 18 \text{ sq. units}$$

17. Since the given curve represented by the equation $y^2 = 4x$ is a parabola



$\therefore$ Required area

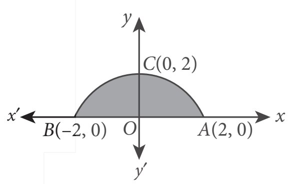
$$= \int_2^4 y\,dx$$

$$= \int_2^4 2\sqrt{x}\,dx = 2\int_2^4 x^{\frac{1}{2}}\,dx$$

$$= 2\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_2^4 = \frac{4}{3}\left[(4)^{\frac{3}{2}} - (2)^{\frac{3}{2}}\right] = \frac{4}{3}(8 - 2\sqrt{2}) \text{ sq. units}$$

18. We have, $y = \sqrt{4 - x^2}$

$\Rightarrow$ $x^2 + y^2 = 4$, which is circle with centre (0, 0) and radius 2 units



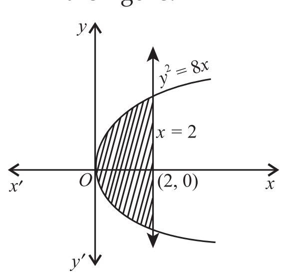
$\therefore$ Required area

$$= \int_{-2}^{2} \sqrt{4 - x^2}\,dx = \left[\frac{x}{2}\sqrt{4 - x^2} + 2\sin^{-1}\left(\frac{x}{2}\right)\right]_{-2}^{2}$$

$$= \left[\left\{\frac{2}{2}\sqrt{4-4} + 2\sin^{-1}\left(\frac{2}{2}\right)\right\} - \left\{\frac{-2}{2}\sqrt{4-(-2)^2} + 2\sin^{-1}\left(\frac{-2}{2}\right)\right\}\right]$$

$$= \left[1\times 0 + 2\times\frac{\pi}{2} + 1\times 0 + 2\times\frac{\pi}{2}\right] = 2\pi \text{ sq. units}$$

19. The rough sketch of the parabola $y^2 = 8x$ and line $x = 2$ is as shown in the figure.

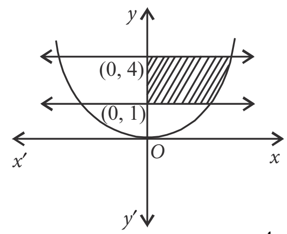


The area of shaded region $= 2\int_0^2 y\,dx = 2\int_0^2 2\sqrt{2x}\,dx$

$$= 4\sqrt{2}\int_0^2 \sqrt{x}\,dx = 4\sqrt{2}\left[\frac{2}{3}x^{3/2}\right]_0^2$$

$$= 4\sqrt{2}\times\frac{2}{3}\left[2^{3/2} - 0\right] = \frac{32}{3}\text{sq.units}$$

20. The rough sketch of the curve $y = 9x^2$, $x = 0$, $y = 1$ and $y = 4$ is as shown in the figure.

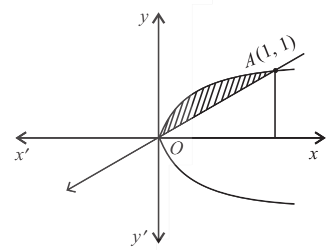


The required area of shaded region $= \int_1^4 x\,dy$

$$= \int_1^4 \frac{\sqrt{y}}{3}\,dy = \frac{1}{3}\left[\frac{2}{3}(y)^{3/2}\right]_1^4$$

$$= \frac{2}{9}\left[4^{3/2} - 1\right] = \frac{2}{9}(8 - 1) = \frac{14}{9}\text{sq.units}$$

21. We have, curves $y = \sqrt{x}$ and $y = x$.



The points of intersection of $y = \sqrt{x}$ and $y = x$ are $O(0, 0)$ and $A(1, 1)$.

The required area of shaded region $= \int_0^1 (y_2 - y_1)\,dx$

where $y_2 = \sqrt{x}$ and $y_1 = x$

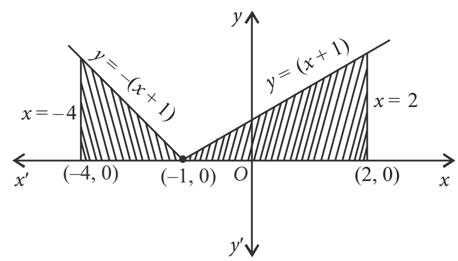
$\therefore$ Required area $= \int_0^1 \left(\sqrt{x} - x\right)dx$

$$= \left[\frac{2x^{3/2}}{3} - \frac{x^2}{2}\right]_0^1 = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}\text{ sq.unit}$$

22. We have, $y = |x + 1|$

$$\therefore \quad y = \begin{cases} -(x+1), & x < -1 \\ (x+1), & x \geq -1 \end{cases}$$

The graph of the curve $y = |x + 1|$ is shown in figure.



$$\therefore \int_{-4}^{2} |x + 1|\,dx = \int_{-4}^{-1} -(x + 1)\,dx + \int_{-1}^{2} (x + 1)\,dx$$

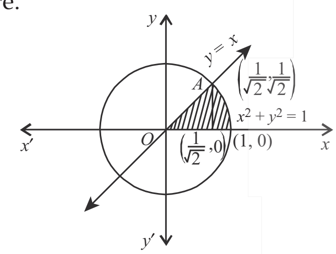
$$= -\left[\frac{x^2}{2} + x\right]_{-4}^{-1} + \left[\frac{x^2}{2} + x\right]_{-1}^{2}$$

$$= -\left[\left(\frac{1}{2} - 1\right) - \left(\frac{16}{2} - 4\right)\right] + \left[\left(\frac{4}{2} + 2\right) - \left(\frac{1}{2} - 1\right)\right]$$

$$= -\left[-\frac{1}{2} - \frac{8}{2}\right] + \left[\frac{8}{2} + \frac{1}{2}\right] = \frac{9}{2} + \frac{9}{2} = 9$$

23. We have, curve $y = \sqrt{1 - x^2} \Rightarrow x^2 + y^2 = 1$ and line $y = x$

The rough sketch of the curve and line $y = x$ is shown in the figure.



The intersection points of line $y = x$ and $x^2 + y^2 = 1$ are $O(0, 0)$ and $A\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$.

$$\therefore \quad \text{Required area} = \int_0^{\frac{1}{\sqrt{2}}} x\,dx + \int_{\frac{1}{\sqrt{2}}}^{1} \sqrt{1 - x^2}\,dx$$

$$= \left[\frac{x^2}{2}\right]_0^{\frac{1}{\sqrt{2}}} + \left[\frac{x}{2}\sqrt{1 - x^2} + \frac{1}{2}\sin^{-1} x\right]_{\frac{1}{\sqrt{2}}}^{1}$$

$$= \frac{1}{4} + \left[\left(0 + \frac{1}{2}\sin^{-1} 1\right) - \left(\frac{1}{2\sqrt{2}}\sqrt{1 - \frac{1}{2}} + \frac{1}{2}\sin^{-1}\frac{1}{\sqrt{2}}\right)\right]$$

$$= \frac{1}{4} + \left[\frac{1}{2} \times \frac{\pi}{2} - \frac{1}{4} - \frac{1}{2} \times \frac{\pi}{4}\right]$$

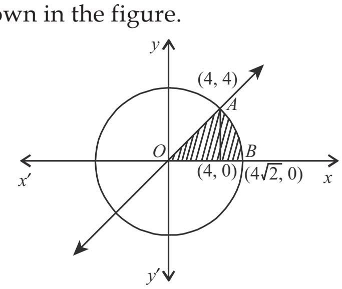
$$= \frac{1}{4} + \frac{\pi}{4} - \frac{1}{4} - \frac{\pi}{8} = \frac{\pi}{8} \text{ sq.unit}$$

24. We have, $x^2 + y^2 = 32$ ...(i)

and $y = x$ ...(ii)

Solving (i) and (ii), the intersection points are $O(0, 0)$ and $A(4, 4)$ in first quadrant.

The rough sketch of the circle $x^2 + y^2 = 32$ and line $y = x$ is shown in the figure.



$$\therefore \quad \text{Required area} = \int_0^4 x\,dx + \int_4^{4\sqrt{2}} \sqrt{32 - x^2}\,dx$$

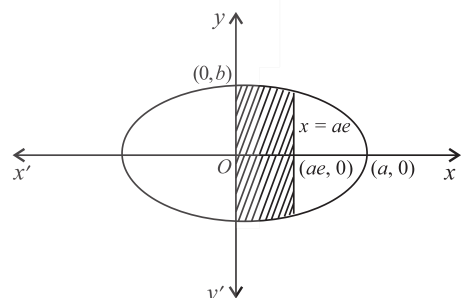
$$= \left[\frac{x^2}{2}\right]_0^4 + \left[\frac{x}{2}\sqrt{32 - x^2} + \frac{32}{2}\sin^{-1}\frac{x}{4\sqrt{2}}\right]_4^{4\sqrt{2}}$$

$$= 8 + \left[\left(0 + \frac{32}{2}\sin^{-1}\frac{4\sqrt{2}}{4\sqrt{2}}\right) - \left(\frac{4}{2}\sqrt{32 - 16} + \frac{32}{2}\sin^{-1}\frac{4}{4\sqrt{2}}\right)\right]$$

$$= 8 + \left[16\sin^{-1} 1 - 8 - 16\sin^{-1}\frac{1}{\sqrt{2}}\right]$$

$$= 8 + \frac{16\pi}{2} - 8 - 16\left(\frac{\pi}{4}\right) = 4\pi \text{ sq.units}$$

25. The rough sketch of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the line $x = 0$ and $x = ae$ is shown in the figure.



The required area of shaded region $= 2\int_0^{ae} y\,dx$,

where $y = \frac{b}{a}\sqrt{a^2 - x^2}$ $\therefore$ Area $= \frac{2b}{a}\int_0^{ae} \sqrt{a^2 - x^2}\,dx$

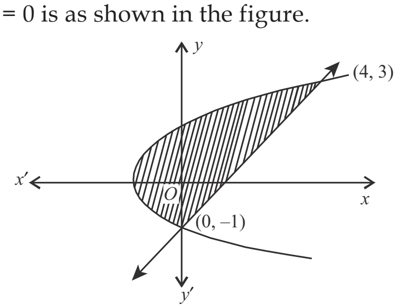
$$= \frac{2b}{a}\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^{ae}$$

$$= \frac{2b}{2a}\left[ae\sqrt{a^2 - a^2e^2} + a^2\sin^{-1}\frac{ae}{a}\right] - 0$$

$$= \frac{b}{a}\left[ae\sqrt{a^2(1 - e^2)} + a^2\sin^{-1} e\right]$$

$$= ab\left[e\sqrt{1 - e^2} + \sin^{-1} e\right] \text{sq.units}$$

26. The rough sketch of the parabola $y^2 = 2x + 1$ and line $x - y - 1 = 0$ is as shown in the figure.



The intersection points of $y^2 = 2x + 1$ and $x - y = 1$ are $(0, -1)$ and $(4, 3)$.

The required area of shaded region

$$= \int_{-1}^{3} (x_1 - x_2)dy$$

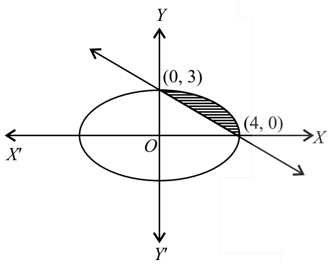
where $x_1 = y + 1$ and $x_2 = \dfrac{y^2 - 1}{2}$

$$\therefore \quad \text{Area} = \int_{-1}^{3}\left[(y+1) - \left(\frac{y^2-1}{2}\right)\right]dy$$

$$= \left[\frac{y^2}{2} + y - \frac{y^3}{6} + \frac{1}{2}y\right]_{-1}^{3} = \left[\frac{y^2}{2} - \frac{y^3}{6} + \frac{3y}{2}\right]_{-1}^{3}$$

$$= \left(\frac{9}{2} - \frac{27}{6} + \frac{9}{2}\right) - \left(\frac{1}{2} + \frac{1}{6} - \frac{3}{2}\right) = \frac{16}{3}\text{sq.units}$$

27. The rough sketch of ellipse $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$ and the line $\dfrac{x}{4} + \dfrac{y}{3} = 1$ is shown in the figure.



$$\therefore \quad \text{Required area} = \int_0^4\left[\frac{3}{4}\sqrt{16 - x^2} - \frac{3}{4}(4 - x)\right]dx$$

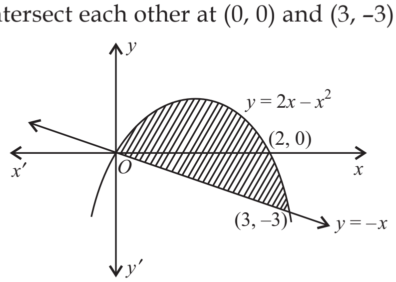
$$= \frac{3}{4}\int_0^4\left(\sqrt{16 - x^2} - 4 + x\right)dx$$

$$= \frac{3}{4}\left[\frac{x}{2}\sqrt{16 - x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4} - 4x + \frac{x^2}{2}\right]_0^4$$

$$= \frac{3}{4}\left[0 + 8\sin^{-1}1 - 16 + 8 - 0\right]$$

$$= \frac{3}{4}\left[8 \times \frac{\pi}{2} - 8\right] = \frac{3}{4}[4\pi - 8] = 3(\pi - 2)\text{ sq.units}$$

28. The curve $y = 2x - x^2$ represents a parabola opening downwards and cutting x-axis at (0, 0) and (2, 0). Clearly, $y = -x$ represents a line passing through the origin and making 135° with x-axis. A rough sketch of the two curves is shown in the figure. The region whose area is to be found is shaded in figure. The two curves intersect each other at (0, 0) and (3, –3).



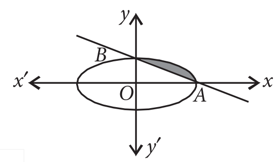
$\therefore$ Required area

$$= \int_0^3\left\{2x - x^2 - (-x)\right\}dx$$

$$= \int_0^3 (3x - x^2)\,dx = \left[\frac{3}{2}x^2 - \frac{x^3}{3}\right]_0^3$$

$$= \frac{27}{2} - \frac{27}{3} = \frac{9}{2}\text{sq.units}$$

29. Required area



$$= \int_0^a\left(\frac{b}{a}\sqrt{a^2 - x^2} - \frac{b}{a}(a - x)\right)dx$$

$$= \frac{b}{a}\int_0^a\sqrt{a^2 - x^2}\,dx - \frac{b}{a}\int_0^a(a - x)\,dx$$

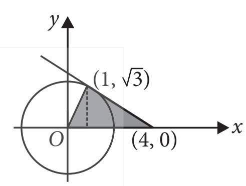
$$= \frac{b}{a}\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a - \frac{b}{a}\left[ax - \frac{x^2}{2}\right]_0^a$$

$$= \frac{b}{a}\left[\frac{a^2}{2}\sin^{-1}1\right] - \frac{b}{a}\left[a^2 - \frac{a^2}{2}\right]$$

$$= \frac{ab}{2}\frac{\pi}{2} - ba\left(\frac{1}{2}\right) = \frac{ab}{4}(\pi - 2)\text{ sq.units}$$

30. The tangent on $x^2 + y^2 = 4$ at $(1, \sqrt{3})$ is $x + \sqrt{3}y = 4$ and equation of normal at $(1, \sqrt{3})$ is $y = x\sqrt{3}$.

$\therefore$ Required area



$$= \int_0^1 x\sqrt{3}\,dx + \int_1^4 \frac{4 - x}{\sqrt{3}}dx$$

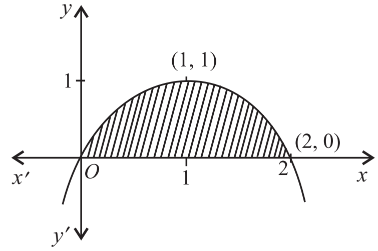
$$= \sqrt{3}\left[\frac{x^2}{2}\right]_0^1 + \frac{1}{\sqrt{3}}\left[4x - \frac{x^2}{2}\right]_1^4$$

$$= \sqrt{3} \times \frac{1}{2} + \frac{1}{\sqrt{3}}\left[4(4-1) - \frac{1}{2}(16-1)\right] = \frac{\sqrt{3}}{2} + \frac{3\sqrt{3}}{2} = 2\sqrt{3}\text{ sq. units}$$

31. We have, $y = 2x - x^2 \Rightarrow -y = x^2 - 2x$

$\Rightarrow \quad -y + 1 = x^2 - 2x + 1 \Rightarrow -(y - 1) = (x - 1)^2$

Clearly it represents a parabola opening downwards whose vertex is (1, 1) and cuts x-axis at (0, 0) and (2, 0). The rough sketch of the curve is given below :



$$\therefore \quad \text{Required area} = \int_0^2 y\,dx$$

$$\therefore \quad = \int_0^2 (2x - x^2)dx$$

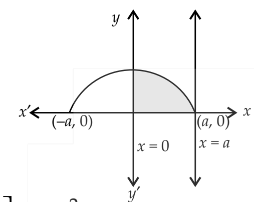
$$= \left[x^2 - \frac{x^3}{3}\right]_0^2 = 4 - \frac{8}{3} = \frac{4}{3}\text{sq.units}$$

32. We have given the equation of curve

$$y = \sqrt{a^2 - x^2} \Rightarrow y^2 = a^2 - x^2$$

$\Rightarrow$ $y^2 + x^2 = a^2$, a circle with centre (0, 0) and radius a

Thus the required area = area of the shaded region



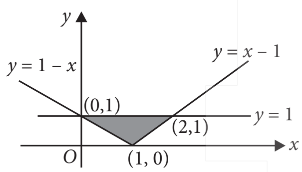
$$= \int_0^a \sqrt{a^2 - x^2}\,dx$$

$$= \left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$$

$$= \left[0 + \frac{a^2}{2}\sin^{-1}(1) - 0 - \frac{a^2}{2}\sin^{-1}(0)\right] = \frac{\pi a^2}{4}$$

33. We have, $y = x - 1$, if $x - 1 \geq 0$

$y = -x + 1$, if $x - 1 < 0$

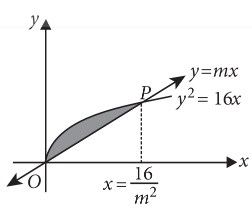


$\therefore$ Required area

$$= \int_0^2 1\,dx - \left[\int_0^1 (1 - x)\,dx + \int_1^2 (x - 1)\,dx\right]$$

$$= [x]_0^2 - \left[x - \frac{x^2}{2}\right]_0^1 - \left[\frac{x^2}{2} - x\right]_1^2 = 2 - \frac{1}{2} - \frac{1}{2} = 1 \text{ sq. unit}$$

34. We have, $y^2 = 16x$, a parabola with vertex (0, 0) and line $y = mx$.



$\therefore$ Required area

$$= \int_0^{16/m^2} (\sqrt{16x} - mx)dx = \frac{2}{3}$$

$$\Rightarrow \left[4 \times \frac{2}{3}x^{3/2} - m\left(\frac{x^2}{2}\right)\right]_0^{16/m^2} = \frac{2}{3}$$

$$\Rightarrow \frac{8}{3} \times \frac{64}{m^3} - \frac{m}{2}\frac{256}{m^4} = \frac{2}{3} \Rightarrow \frac{1}{m^3}\left[\frac{512}{3} - 128\right] = \frac{2}{3}$$

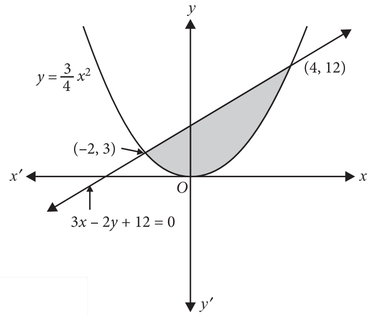
$\Rightarrow$ $m = 4$

35. Given equations are $y = \dfrac{3x^2}{4}$ ...(i)

and $3x - 2y + 12 = 0$ $\Rightarrow$ $y = \dfrac{3x + 12}{2}$ ...(ii)

Solving (i) and (ii), we get

$$\frac{3x^2}{4} = \frac{3x + 12}{2}$$



$\Rightarrow$ $x^2 - 2x - 8 = 0 \Rightarrow (x + 2)(x - 4) = 0$

$\Rightarrow$ $x = -2, 4$

$\therefore$ Required area $= \displaystyle\int_{-2}^{4} \left(\frac{3x + 12}{2} - \frac{3}{4}x^2\right)dx$

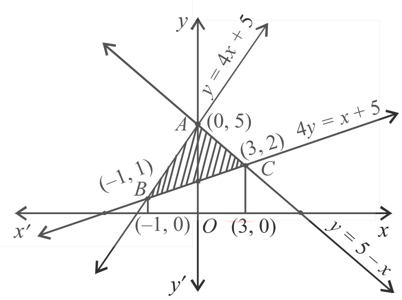
$$= \left[\frac{3}{4}x^2 + 6x - \frac{x^3}{4}\right]_{-2}^4$$

$$= \left[\frac{3 \times 16}{4} + 6 \times 4 - \frac{64}{4}\right] - \left[\frac{3}{4} \times 4 - 6 \times 2 + \frac{8}{4}\right]$$

= 27 sq. units.

36. We have, $y = 4x + 5$, $y = 5 - x$ and $4y = x + 5$

The rough sketch of the lines is shown in the figure.



Here, equation of line AB is $y = 4x + 5$,

equation of line AC is $y = 5 - x$ and

equation of line BC is $y = \dfrac{x + 5}{4}$

The intersection point of line AB and AC is at $A(0, 5)$. Similarly, the intersection point of line AB and line BC is at $B(-1, 1)$ and the intersection point of line AC and BC is at $C(3, 2)$.

$\therefore$ Required area $= \displaystyle\int_{-1}^{0} (4x + 5)dx + \int_0^3 (5 - x)dx - \int_{-1}^{3} \frac{x + 5}{4}dx$

$$= \left[2x^2 + 5x\right]_{-1}^0 + \left[5x - \frac{x^2}{2}\right]_0^3 - \frac{1}{4}\left[\frac{x^2}{2} + 5x\right]_{-1}^3$$

$$= 0 - (2 - 5) + \left(15 - \frac{9}{2}\right) - \frac{1}{4}\left[\left(\frac{9}{2} + 15\right) - \left(\frac{1}{2} - 5\right)\right]$$

$= 3 + \frac{21}{2} - \frac{1}{4}\left[\frac{39}{2} + \frac{9}{2}\right]$

$= 3 + \frac{21}{2} - 6 = \frac{21}{2} - 3 = \frac{15}{2}$ sq.units

37. Given, $y = |x + 1| + 1$

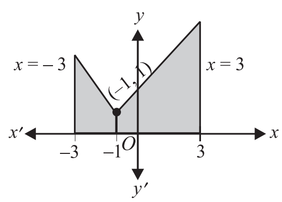
$\therefore \quad y = \begin{cases} x+2 & \text{if } x \geq -1 \\ -x & \text{if } x < -1 \end{cases}$

We now draw the lines : $y = 0$, $x = 3$, $x = -3$ and

$y = x + 2$ if $x \geq -1$ ...(i)

$y = -x$ if $x < -1$ ...(ii)

Lines (i) and (ii) intersect each other at $(-1, 1)$



$\therefore$ Required area $= \int_{-3}^{-1} (-x)dx + \int_{-1}^{3} (x+2)dx$

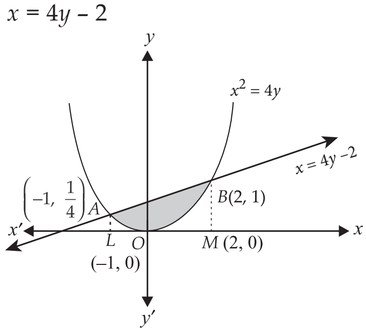
$= -\left[\frac{x^2}{2}\right]_{-3}^{-1} + \left[\frac{x^2}{2} + 2x\right]_{-1}^{3}$

$= -\frac{1}{2}(1-9) + \frac{1}{2}(9-1) + 2(3+1)$

$= 4 + 4 + 8 = 16$ sq. units.

38. The given curve is $x^2 = 4y$ ...(i)

and line is $x = 4y - 2$ ...(ii)



Solving (i) and (ii), we get $(x + 2) = x^2$

$\Rightarrow \quad x^2 - x - 2 = 0 \Rightarrow (x - 2)(x + 1) = 0 \Rightarrow x = 2, -1$

Thus the points of intersection of the given curve and line are $A\left(-1, \frac{1}{4}\right)$ and B (2, 1)

$\therefore$ Required area

$= \int_{-1}^{2} \left(\frac{x+2}{4}\right)dx - \int_{-1}^{2} \frac{x^2}{4}dx = \int_{-1}^{2}\left(\frac{x}{4} + \frac{1}{2} - \frac{x^2}{4}\right)dx$

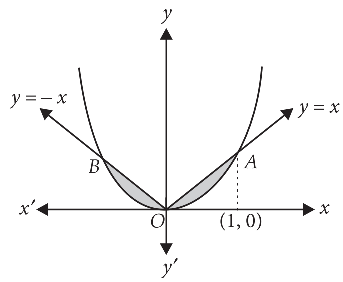
$= \frac{1}{4}\left[\frac{x^2}{2}\right]_{-1}^{2} + \frac{1}{2}[x]_{-1}^{2} - \frac{1}{4}\left[\frac{x^3}{3}\right]_{-1}^{2}$

$= \frac{1}{8}[4-1] + \frac{1}{2}[2+1] - \frac{1}{12}[8+1]$

$= \frac{3}{8} + \frac{3}{2} - \frac{3}{4} = \frac{3}{2}\left[\frac{1}{4} + 1 - \frac{1}{2}\right] = \frac{3}{2}\left[\frac{3}{4}\right] = \frac{9}{8}$ sq. units

39. The given curves are $y = x^2$ ...(i)

$y = |x| = \begin{cases} x, \text{ if } x \geq 0 \\ -x, \text{ if } x < 0 \end{cases}$ ...(ii)



Their points of intersection are $A(1, 1)$, $O(0, 0)$ and $B(-1, 1)$.

$\therefore$ Required area

$= 2\int_{0}^{1}(x - x^2)dx = 2\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = 2\left[\frac{1}{2} - \frac{1}{3}\right] = \frac{1}{3}$ sq.unit.

40. We have curves, $y = \frac{1}{\sqrt{3}}x$ ...(i)

and $x^2 + y^2 = 16$ ...(ii)

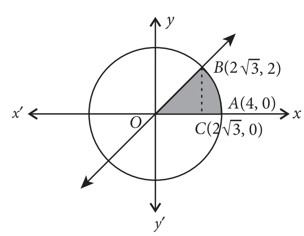
Curves (i) and (ii) intersect each other at $(2\sqrt{3}, 2)$ and $(-2\sqrt{3}, -2)$.

$\therefore$ Required area = Area of region $OBAO$

= area ΔOBC + area of region $BCAB$

$= \int_{0}^{2\sqrt{3}} \frac{x}{\sqrt{3}}dx + \int_{2\sqrt{3}}^{4} \sqrt{16 - x^2}dx$

$= \left[\frac{x^2}{2\sqrt{3}}\right]_0^{2\sqrt{3}} + \left[\frac{x}{2}\sqrt{16 - x^2} + \frac{16}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_{2\sqrt{3}}^{4}$



$= 2\sqrt{3} + 8\left(\frac{\pi}{2}\right) - 2\sqrt{3} - \frac{8\pi}{3}$

$= \frac{12\pi - 8\pi}{3} = \frac{4\pi}{3}$ sq.units